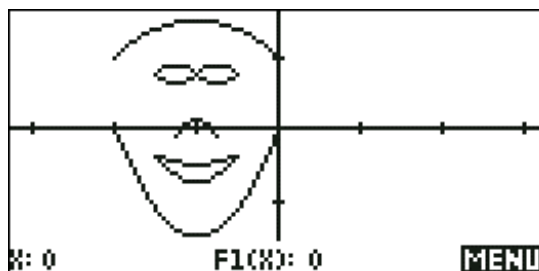


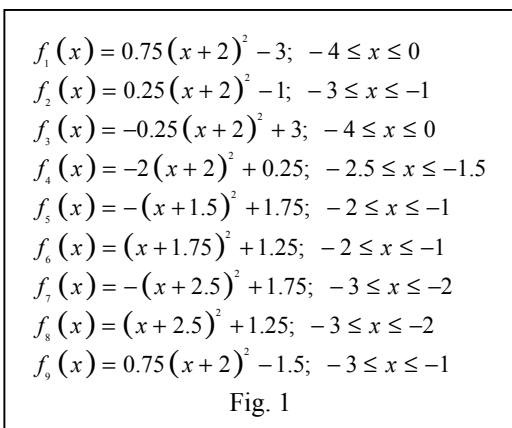
The Artistry of Mathematics

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The introduction of the concept of the domain, perhaps in Introductory Calculus, is an ideal opportunity to use graphical calculators to explore the creative side of your students. Using restricted domains, with only quadratics and lines, it is quite easy to draw pictures such as the one shown right.



Working in this case with a very able class, I began the activity by displaying the image above as a 'teaser' and then explaining that it was entirely built using quadratics. I showed the students the equations that I had used (see fig. 1) and explained how the calculator could be used to display restricted domains by dividing by the domain in the form of a logical expression. Although the examples in this article are done using a Hewlett-Packard HP39G, the same process can be used on any of the commonly available graphical calculators.



For those who have not come across this trick before, the procedure is to divide the function by the domain required. For example, to graph $f(x) = x^2; x < 1$ you would enter the function as $F1(X)=X^2/(X<1)$.

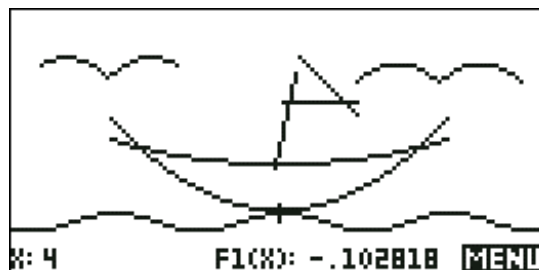
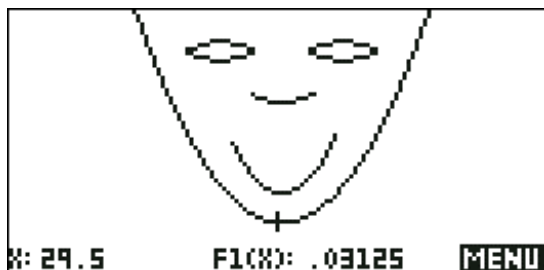
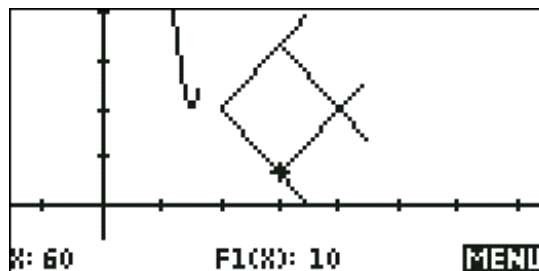
The logical expression $(X<1)$ evaluates to 1 (true) within the domain and zero (false) outside it. Thus dividing by the logical expression means that the function will be unaffected within the domain but undefined outside it and for the section(s) where it is undefined it will not be graphed. The domains used can be made as complex as desired by using the AND and OR operators.

For example:



The next stage of the activity was to challenge the group to produce their own images. They were given a week to work and two restrictions were placed on their work. The first was that they must use no more than 10 functions to form the image and the second that they must restrict themselves to quadratics and linear functions. The incentive of a chocolate prize was added in order to start the students off with enthusiasm! Some of the results are shown below and on the next page. As can be seen, the complexity of the resulting images varied greatly.





The more observant reader will have noticed that the last image, created by Min Kang, includes an ‘ocean’ made from a trigonometric graph. Although this was a technical infraction of the rules the class voted to include the image. The winner of the competition was the pair of penguins created by Ruth Mills, with Min’s boat in second place.

Ruth’s functions were quite a bit more complex than I had expected. Frustrated by the restriction to ten functions, she had clearly been experimenting extensively and had used combinations of domains in order to use the same function to form different parts of the two penguins. Being unfamiliar with the OR command, she had combined domains by adding them. Using this method the domain $(-1 \leq x \leq 2) \cup (3 \leq x \leq 5)$ became $((x \geq -1 \text{ AND } x \leq 2) + (x \geq 3 \text{ AND } x \leq 5))$. The effect in this case was functionally the same. Some of her final functions were so involved that they would not fit onto the width of this page when I tried to type them in!

This is an activity that you and your students will find very enjoyable and you may find yourself pleasantly surprised by the creativity of your class. It can be coped with by students of lower to middle ability yet offers a challenge to the more able student. It can also easily be re-visited and extended to include other functions as they become available during the year.