

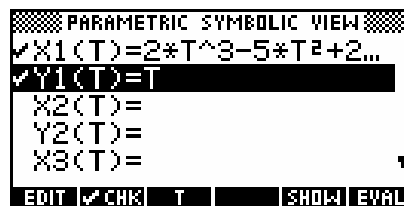
Vectors in the Parametric Aplet

The Parametric aplet can be used to visually display vector motion in one and two dimensions.

Example 1

A particle P is moving in a straight line. Its velocity v (in ms^{-1}) at any time t (in seconds, $t > 0$) is given by $v(t) = 2t^3 - 5t^2 + 2t - 3$. Illustrate its motion during the first 2.5 seconds.

Enter the motion equation from (v) as $X(T)$ and enter $Y(T)=T$. The only purpose of this second equation is to move the particle up the y axis as it traces out its path, thereby making it easier to view.

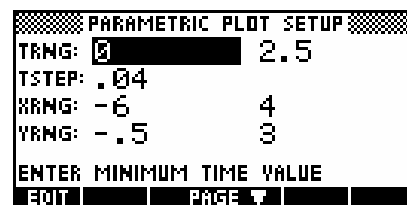


Changing to the NUM view lets me scroll through the first three seconds of movement, allowing me to choose a good scale for the x axis.

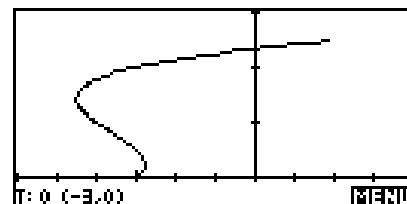
T	X1	Y1
1.25	-4.40625	1.25
1.5	-4.5	1.5
1.75	-4.09375	1.75
2	-3	2
2.25	-1.09125	2.25
2.5	2	2.5

2.5
ZOOM | BIG | DEFN

I'm interested in the first $2\frac{1}{2}$ seconds only, so I'll also restrict TRng to 0 to 2.5. Using $Y(T)=T$ for this TRng means the y values will also range from 0 to 2.5. Maximizing visibility of this range of values is the reason for setting YRng to be -0.5 to 3 in PLOT SETUP. The range for the x axis is chosen from the values shown in the NUM view. The value of TStep is carefully chosen so that when the motion plots, the speed is slow enough to show its progress.



The graph makes it plain that it doubles back twice in the first two seconds. The really nice part about this method though is that the motion can be *seen* on the screen. As the particle slows down near the turning points it does so on the screen. As it accelerates in the final section you can see this on the screen too. Obviously this can't be seen on this page, and I recommend you try this for yourself.



Example 2

Two ships are traveling according to the vector motions given below, where time is in hours and distance in kilometers. Illustrate their motion during the first ten hours.

$$\text{Ship A: } \tilde{x}_A = \begin{pmatrix} -100 \\ 500 \end{pmatrix} + \begin{pmatrix} 20 \\ -30 \end{pmatrix} t$$

$$\text{Ship B: } \tilde{x}_B = \begin{pmatrix} 200 \\ 400 \end{pmatrix} + \begin{pmatrix} -15 \\ -20 \end{pmatrix} t$$

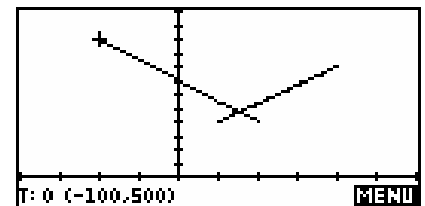
Enter the equations of motion as shown right.

```
PARAMETRIC SYMBOLIC VIEW
✓X1(T)=-100+20*T
✓Y1(T)=500-30*T
✓X2(T)=200-15*T
✓Y2(T)=400-20*T
X3(T)=
EDIT | ✓CHK | T | SHOW | EVAL
```

Now change to the PLOT SETUP view and set the axes to suitable values. Possible values are shown below.

PARAMETRIC PLOT SETUP	PARAMETRIC PLOT SETUP
TRNG: 0 10	XTICK: 50 YTICK: 50
TSTEP: .2	✓SIMULT _INV. CROSS
XRNG: -200 300	✓CONNECT _LABELS
YRNG: -100 600	✓AXES _GRID
ENTER MINIMUM TIME VALUE	ENTER HORIZONTAL TICK SPACING
EDIT PAGE ↓	EDIT ▲ PAGE

Now press PLOT to see the ships paths appear. As with the previous example, the value of TStep is chosen to allow visible motion. As the graph appears it can be seen that the ships do not collide, even though the final plot may make it appear that they do.



To find the distance between the two ships at any time t , you can enter the equation

$F1(X) = \sqrt{(X1(X) - X2(X))^2 + (Y1(X) - Y2(X))^2}$ into the Function applet. Note that the active variable must be an X in the Function applet instead of the T of the Parametric applet but you can still refer to the Parametric functions $X1$ and $Y1$ even within the Function applet. Graphing this function in the PLOT view of the Function applet will allow you to find its minimum value. In this case the minimum separation of 13.74 km is achieved at $t=8.68$ hrs.

