

Using the HP 39gs & HP 49gs effectively

by Colin Croft
www.hphomeview.com
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1 Resetting the calculator

As with computers, the hp 39gs and hp 40gs sometimes crash due to the very complex and flexible operating system they use. If you find that the calculator is beginning to behave strangely, or is locking up then there are a number of ways to deal with this.

1.1 Soft reboot (Keyboard)

Pressing ON+SK3 will perform a soft reboot and will not cause any memory loss. Hold down the ON key and, while still holding it down, press the third screen key from the left. The calculator will very briefly display a boot screen and then redisplay the HOME view, with the Function aplet as the active one.

1.2 Paper clip reboot

If you find that the calculator locks up so completely that the keyboard will not respond then a method of reset is provided below which is independent of the keyboard. This shouldn't happen but it is important to know how to deal with it in case it happened during a test or an exam.

1.3 Soft reboot (Hardware)

Very rarely, a calculator may lock up to the point that the keyboard no longer responds. On the back of the calculator is a small hole. Poke a paper clip or a pin into this hole and press gently on the switch inside. This briefly interrupts the power supply and, to the calculator, is exactly equivalent to a soft reboot as outlined in the previous section. Normally it should not only unlock the calculator but retain your data intact. However, it must be pointed out that a problem so severe that it has locked up the keyboard may be so bad that it has corrupted the memory anyway. If so then you may find that you see the "Memory Clear" message when it reboots. If you find that it keeps locking up repeatedly then you should perform a hard reboot as described in the next section. If you have important data on the calculator then you might try to save it to a PC before doing this since a hard reboot will wipe all user memory.

1.4 Hard reboot (with loss of memory)




To completely reset the calculator's memory back to factory settings press ON+SK1+SK6. (SK1="screen key 1") When doing this, don't press them all at once; hold down the ON key and, while still holding it down, press the first and then the last screen keys. Release them in the opposite order. Don't release SK1 and SK6 together - release SK6, then SK1, then ON.

This type of reset will always cause complete loss of data. If you find that the screen fills with garbage, or if the calculator's in-built diagnostic routine starts to run, then it is just that you have not released them in the right order. Simply try again.

1.5 Odd behaviour?

Locked Calculator? Won't turn on? Turns on briefly and then immediately off?

Try the following:

- Try a soft reboot using ON+SK3.
- Try a paperclip in the hole in the back (gently).
- Is it possible that the screen is blank because the contrast is turned to extremely light? Try ON and  to make the screen darker. (Hold down ON and press  repeatedly.) Use ON and  to lighten the screen.
- Change the batteries for new ones. Try this more than once because sometimes batteries are dead right off the shelf if they've been in the shop too long. Try replacing the backup battery too. Make SURE you've put the batteries in right way around. Putting them in reversed may seriously damage the calculator.
- Take the batteries out, including the round backup battery. Press and hold the ON button for 2 minutes to remove any possible remaining power from the internal capacitors. Leave the calculator overnight and re-insert fresh batteries.
- If this happened when running a game or applet that you've downloaded from the internet then consider that this may be the source of the problem. Backup anything that you want to keep to the PC and do a full reboot to restore factory settings.
- Try a full reboot using ON+SK1+SK6.

Some less likely options:

- Did the paperclip feel funny when you inserted it? There should be a very subtle sensation of 'pressing a button' as the paperclip shorts out the batteries momentarily. Could you have accidentally bent the shorting contact last time you used a paperclip so that it is permanently shorting out?
- Check the battery compartment. Are any of the metal contacts broken or out of place? Are the batteries firmly in contact with all of them? Is anything out of place? Do any of the batteries or the metal contacts show corrosion? If so, clean them carefully, being careful not to get moisture inside the calculator.
- Have you recently dropped the calculator or spilled liquid on it? If so, this is not good. It is probably permanently dead. Did you need a very expensive paperweight?
- Check the USB and Serial ports at the top of the calculator. Do they have anything wedged in them courtesy of a young child perhaps?

2 Using the HOME view

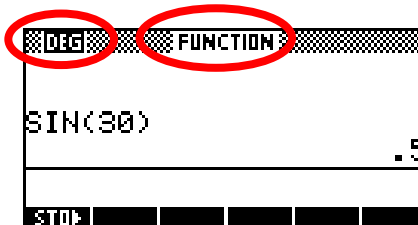
2.1 Angle and Numeric settings

It is critical to efficient use of the hp 39gs and hp 40gs that the angle and numeric settings are understood.

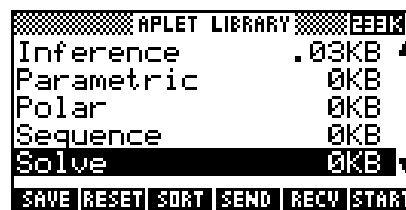
On the hp 39gs and hp 40gs, when you set the angle measure or the numeric format in the MODES view, it applies both to the aplet and to the HOME view. However, this setting applies *only* to the currently active aplet (the one highlighted in the APLET view).

This means that if you change active aplet then these settings may change also, not only for the aplet but in the HOME view too.

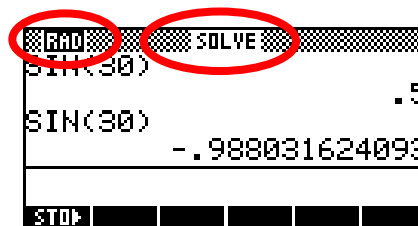
For example, suppose you have been performing trig calculations in the HOME view with the Function aplet being currently active, and have set the angle measure to DEG.



If you were to now change to the Solve aplet in order to solve an equation then the settings would revert to those of the Solve aplet; probably be radians unless you had also changed those as well.



Radian measure is the default for all aplets and thus also for HOME unless you change it. Performing exactly the same calculation in HOME would now give a different result, as shown right.



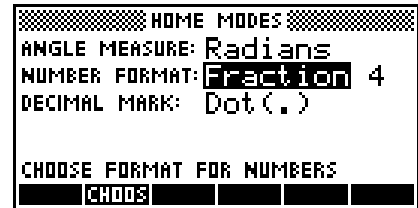
Although this may seem to be a strange way of doing things, there is actually a very good reason for it. On the original first model, the hp 38g, the settings for the HOME view were independent from those of the aplets and it caused a number of users to have difficulties because they would find values of (say) the intersection of two trig graphs in radian mode in the Function aplet and then use them in HOME, which on the 38g defaulted to degree mode, and find that they weren't working correctly.

The only drawback of synchronizing the settings of the HOME and aplet is that you might change aplets and forget that it may also change the HOME view settings. For this reason, the name of the active aplet is shown at the top of the HOME view as a reminder.

Although there's no reminder on the screen, the same idea is used for numeric settings. If you are in the Solve aplet and set it to Fixed 4 then when you change into another aplet you'll find that it will have reverted to the default 'Standard' setting.

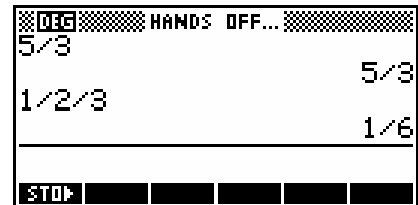
2.2 Fractions on the hp 39gs and hp 40gs

The Fraction setting can be a little deceptive. Begin by selecting Fraction in the MODES view, leaving the accompanying number as the default value of 4.



Most calculators have a fraction key, often labeled a^b/c , that allows you to input, for example, $1\frac{2}{3}$ as $1\rightarrow 2\rightarrow 3$ or something similar. What these calculators usually won't do is allow you to mix fractions and decimals. A calculation such as $1\frac{2}{3} + 3.7$ will usually give a decimal result: most calculators will not attempt to convert the 3.7 into a fraction. The reason for this is that while some decimals like 0.25 are easy to convert to a fraction, others, such as recurring ones, are not so easy. Most calculators opt for the easy option of switching to a decimal answer in any mixture of fractions and decimals. When making the hp 39gs and hp 40gs HP took a very different approach. Once you select Fraction mode, all numbers become fractions - including any decimals.

The first point to remember is that there is no provision for inputting mixed fractions such as $1\frac{2}{3}$. Fractions are entered using the divide key and, while the calculator is quite happy with improper fractions such as $5/3$, it correctly interprets $1/2/3$ as $\frac{1}{2}$ divided by 3 and gives a result of $1/6$. The solution to this is simply to enter mixed fractions as $(1+2/3)$.



Calculator Tip

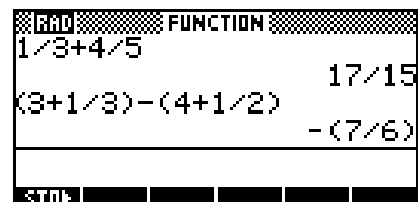
You need to be careful with brackets or else “order of operations” problems may occur, such as $1\frac{2}{3} * \frac{1}{5}$ being interpreted as $1+(2/3*1/5)$ rather than as it should be: $(1+2/3)*1/5$.

When in doubt, use brackets for mixed fractions.

Some examples are... (using a setting of Fraction 4 or higher)

$$\frac{1}{3} + \frac{4}{5} = \frac{17}{15}$$

$$3\frac{1}{3} - 4\frac{1}{2} = -1\frac{1}{6}$$



The second point to remember involves the method the hp 39gs and hp 40gs use when converting decimals to fractions, which is basically to generate (internally and unseen by you) a series of continued fractions which are *approximations* to the decimal entered. The final fractional approximation chosen for display is the first one found which is ‘sufficiently close’ to the decimal. Look up ‘continued fractions’ on the web or in a textbook if you don’t know what these are.

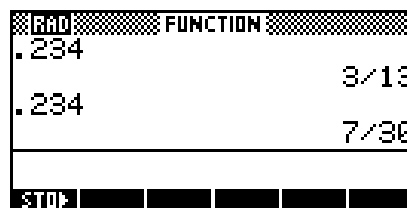
The trap lies in what constitutes ‘sufficiently close’, and this is determined by the ‘4’ in Fraction 4. Very roughly explained, the calculator will use the first fraction it finds in its process of approximation which matches the decimal to that number of significant digits.

For example, a setting in the MODES view of...

Fraction 1

changes 0.234 to $\frac{3}{13}$

which is actually 0.2307692...
(matching to at least 1 significant figures.)



Fraction 2

changes 0.234 to $\frac{7}{30}$

which is actually 0.2333333...
(matching to at least 2 significant figures.)



Fraction 3

changes 0.234 to $\frac{11}{47}$

which is actually 0.2340425...
(matching to at least 3 significant figures.)

Fraction 4

changes 0.234 to $\frac{117}{500}$ (or $\frac{234}{1000}$)

which is exactly 0.234
(thus finally matching to the required 4 significant figures.)

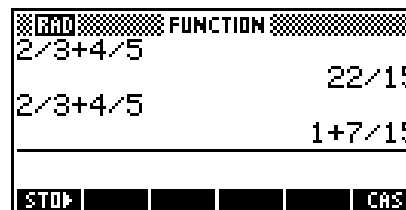
Essentially, the value of 4 in Fraction 4 affects the degree of precision used in converting the decimal to a fraction. As was said earlier, the calculator will use the first fraction it finds in its process of approximation which matches the decimal to that number of significant digits.

The Fraction setting is thus far more powerful than most calculators but can require that you understand what is happening. It should also be clear now why a special fraction button was not provided: the ‘fractions’ are never actually stored or manipulated as fractions at all!



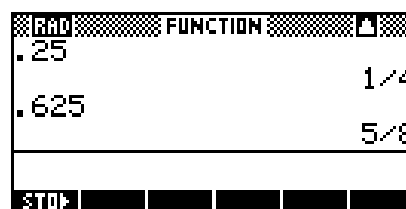
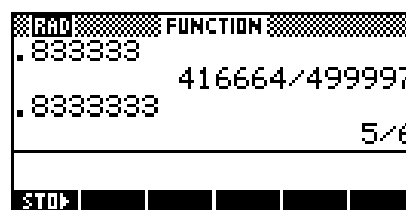
As you can see above right, a setting of Fraction 4 produces a strange (but actually correct) result for 0.666, while adding one more 6 (to take the decimal beyond 4 d.p.) will give the desired result of $2/3$. In other words, so long as you understand the approach taken by the hp 39gs and hp 40gs it is capable of producing results which are closer to what was probably intended by the user in entering 0.66666.

You may have noticed that all the results so far have been improper fractions. For example the first calculation shown right gives the answer as $22/15$ rather than $1\frac{7}{15}$. The fraction setting of Mixed Fraction is essentially the same but answers are given as mixed fractions instead of improper fractions, as shown.



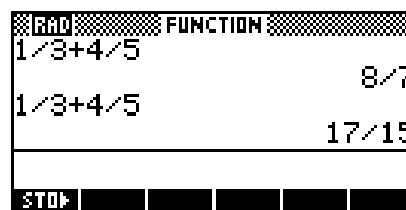
If you want to use the Fraction setting to convert decimals to fractions, here are some tips...

- if converting a recurring decimal to a fraction, then make sure you include at least one more digit in the decimal than the setting of Fraction in MODES. As you can see right, failing to include enough decimal places does not produce the desired result.
- if you are converting an exact decimal to a fraction, then set a Fraction n value of at least one more than the number of decimal places in the value entered. Both examples in the third screen shot to the right were done at Fraction 6.



Not understanding the significance of the setting of Fraction can produce some unfortunate effects. For example, at Fraction 2, the value of 123.456 becomes 123, with the 0.456 dropped entirely.

An example of this is shown right. If you use a setting of only Fraction 2 to perform the calculation shown, you will find to your amazement that $1/3 + 4/5 = 8/7$, whereas using Fraction 6 gives the correct answer.

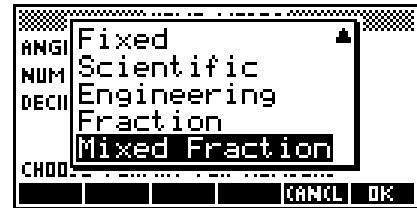


The reason for this ‘error’ is that the $1/3$ and $4/5$ were converted to decimals and added to give 1.133333.... This was converted back to a fraction using Fraction 2 to give $8/7$ (1.1428..).

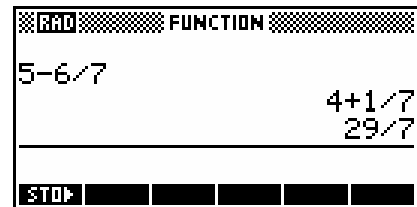
This may seem odd but it does match sufficiently closely in Fraction 2 to be accepted.

Generally it is not a good idea to go below the default setting of Fraction 4. In fact, a Fraction 6 setting tends to be more reliable.

A new feature of the hp 39gs and hp 40gs is the setting of Mixed Fraction in the MODES view.



The results of this new setting can be seen in the image to the right. Using the setting of Mixed Fraction the result is $4+1/7$ ($4\frac{1}{7}$) whereas the answer of $29/7$ is obtained using the old Fraction setting.



Calculator Tip

If you scroll back through the History and re-use a result such as the $4+1/7$ shown above then don't forget to put brackets around it to ensure that no 'order of operations' errors occur.

3 Tips on choosing axes in the Function applet

3.1 Use the default axes

Try plotting the function on the default axes. You may find that enough of the function is showing to give you a rough idea of how to adjust them to display it better. Remember that **ZOOM** can work on either axis or on both. When in doubt, zoom out rather than in.

3.2 Use the NUM view

The NUM view can be very helpful. Try changing to NUM SETUP and setting the value of NumStep to 5 or even 10. Now scroll through the NUM view and look at what is happening to the F(X) values. Look for two things.

- Firstly, where is the function most active? For what domain on the x axis is it changing rapidly both up and down? This is likely to be the domain you are most interested in.
- Secondly, what is the range? What sort of values will you need to display on the y axis? Change to the PLOT SETUP view and set what you think may be appropriate axes. From those you can PLOT and then zoom in or out.

3.3 Use Auto Scale

If the graph is part of a test or an examination then the wording of the question will often give a clue as to what x axis domain you should work with. You can then use Auto Scale.

Auto Scale can be used to get a first approximation to a good set of axes. To do this you must choose your x axis domain first. Use your knowledge of what the function might look like, perhaps together with a quick scroll through the NUM view, to get an idea of what section of the x axis is important.

3.4 An example

For example, suppose a company's profit is modeled by the equation $P(x) = \frac{-100}{x} + 50e^{-0.025x}$, where x is the number of items manufactured. Suppose that from the context of the question it is clear that we are mainly interested in the domain $0 \leq x \leq 100$.

After entering the equation into the SYMB view, change into the NUM SETUP view and set the numeric scale to start at zero and increment in steps of 10 (or 5 if you want more details).

FUNCTION NUMERIC SETUP	
NUMSTART:	0
NUMSTEP:	10
NUMTYPE:	Automatic
NUMZOOM:	4
CHOOSE TABLE FORMAT	
CHDS	PLOT

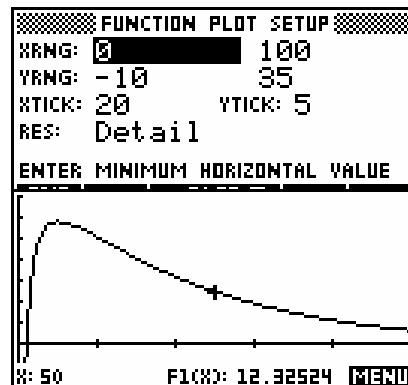
Change into the NUM view and scroll through the window from zero to 100. As you do so, take note of the values that the function takes. From the display it seems that the function peaks around $y=30$ and then declines steadily.

X	F1		
0	UNDEF		
10	28.94004		
20	25.32653		
30	20.28499		
40	15.89397		
50	12.32524		

0

ZOOM	BIG	DEFN
------	-----	------

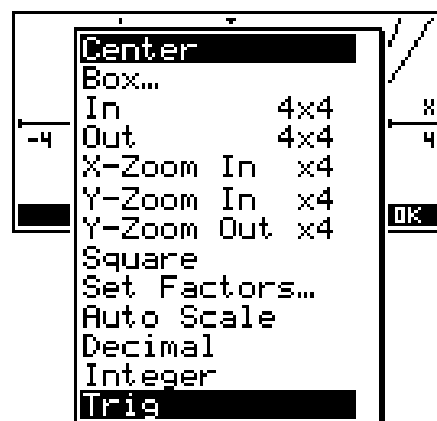
Change into the PLOT SETUP view and enter an x axis of 0 to 100 with a 'tick' value of 20, and a y axis of -10 to 35 with a tick value of 5.



The result of this is a PLOT view as shown right. This would be ideal for answering questions on the domain stated.

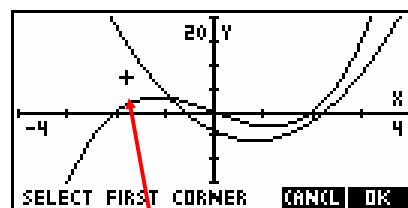
3.5 Using ZOOM

On the menu bar is **ZOOM**. Pressing the key under **ZOOM** pops up a new menu, shown right.



The In/Out options zoom in or out by adjusting the scales by the factor shown. The default factor is 4 for both axes but this can be adjusted through the *Set Zoom Factor* option later in the menu. The most useful settings are either 5x5 or 2x2 as these are more likely to preserve nice scales. Unfortunately the default is 4x4.

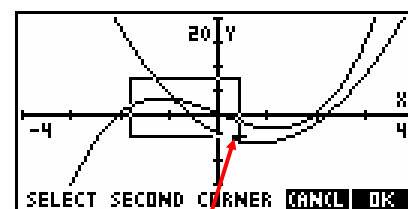
Zoom Box is the most useful of the **ZOOM** commands. When you choose this option a message will appear at the bottom of the screen asking you to *Select first corner*.



The cursor is positioned at one corner of a

If you use the arrow keys to move the cursor to one corner of a rectangle containing the part of the graph you want to zoom into and then press ENTER, the message will change to *Select second corner*.

As you move the cursor to a position at the diagonally opposite corner of a rectangle, the selection box will appear on the screen.

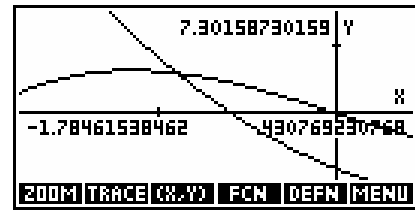


...and then at the other corner.

Pressing **OK** expands the box to fill the screen.

You'll notice that the scale has been disrupted so that the labels are no longer very helpful. PLOT SETUP would give better end points for the axes or let you switch off the labels option. Alternatively you could use the MODES view to set two decimal places.

Rather than doing any of these however, scroll down the **ZOOM** menu to a new option of *UnZoom*. This option puts the screen back to the way it was before you did the **ZOOM**.



4 Using Function effectively in your teaching

4.1 Composite functions

The Function applet is capable of dealing with composite functions such as $f(x+2)$ or $f(g(x))$ in its SYMB view. The **EVAl** and **SHOW** keys are particularly helpful with this.

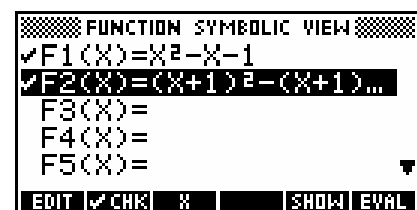
For example, if we define $F1(x) = x^2 - 1$ and $F2(x) = \sqrt{x}$, then we can use these in our defining of $F3(x)$, $F4(x)$. See the screen shot on the left below.

If the highlight is now positioned on each of these in turn, and the **EVAl** key pressed then the substitution is performed. The result is shown in the right hand snapshot.

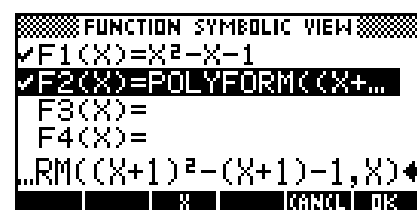


Notice that the calculator is smart enough to realize in $F3(x)$ that $(\sqrt{x})^2 - 1$ is the same as $x - 1$, although not, unfortunately, smart enough to keep track of the implications for the domain, which are that $F3(x)$ should be defined only for non-negative x .

There is a limit to this however. If you define $F1(x) = x^2 - x - 1$ and then $F2(x) = F1(x+1)$, then the **EVAl** routine will not simplify $(x+1)^2 - (x+1) - 1$ to $x^2 + x - 1$.



On the other hand there is a way to further simplify the expression. **EDIT** the result and enclose it with the POLYFORM function as shown right, adding a final 'X' as shown, then highlight it and press **EVAl**. The calculator will expand the brackets and gather terms.



Calculator Tip

These functions can all be graphed but the speed of graphing is slowed if you don't press **EVAl** first. This is because the *composite* function is internally re-evaluated for each point graphed. The hp 39gs and hp 40gs are fast enough that the result is still satisfactory but if you have an old 39g or 40g they are slowed to the point of being unusable.

Evaluating the function may also hide the domain and for this reason it is sometimes best to leave the evaluation undone.

For example, if $F1(X) = X^2$ and $F2(X) = \sqrt{X}$ then $F3(X) = F1(F2(X))$ will show the correct domain of $x \geq 0$ for both $F2(X)$ and $F3(X)$ in the NUM view. Pressing **EVAL** will destroy this.

X	F1	F2	F3
.3	.09	UNDEF.	UNDEF.
.2	.04	UNDEF.	UNDEF.
.1	.01	UNDEF.	UNDEF.
0	0	0	0
.1	.01	.3162278	.1
.2	.04	.4472136	.2

ZOOM | BIG | DEFN

4.2 Using functions in the HOME view

Once functions have been defined in the SYMB view of the Function applet, they can be reused in the HOME view (and indeed in any other applet!).

For example suppose you needed to find some exact points ($x = 0, 1, 2$ and 3) for a graph of $f(x) = \frac{2x}{(x-2)}$ when doing a hand sketch of it.

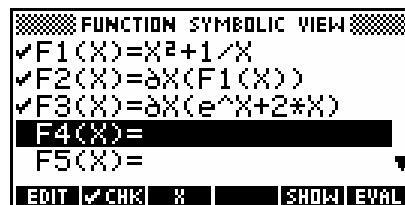
Type its definition into the Function applet SYMB view, switch to the HOME view and then simply type $F1(0)$ and press ENTER. The function will be evaluated for $x = 0$. Similar results can be obtained for $F1(1)$, $F1(2)$ and $F1(3)$. Notice the error message for $x = 2$ caused by a divide by zero.



A simpler way to get function values is to changing into the NUM view and use its tabular listing.

4.3 Differentiating

There are different approaches that can be taken to differentiating, most of which are best done in the SYMB view of the Function applet.



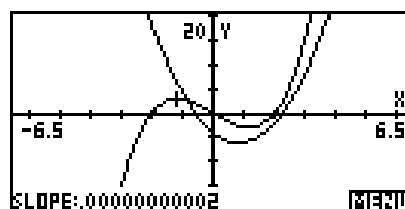
The function can be already defined in the SYMB view of the Function applet as shown in functions $F1(X)$ and $F2(X)$ in the screen shot above. Alternatively it can be directly entered into the brackets as shown in function $F3(X)$.

Algebraic differentiation is most easily handled in the SYMB view of Function. The best method is to define your function as $F1(X)$ and its derivative as $F2(X)$. Doing your differentiation in the Function applet is much easier and offers the additional advantage of being able to graph the two functions.



4.4 Retaining calculated values

When you find an extremum or an intersection, the point is remembered until you move the cursor even if it is not actually on a value that would normally be accessible for the scale you have chosen.



For example, if you find an intersection and then immediately return to the **FCN** menu and choose *Slope*, the slope calculated will be for the intersection just found rather than for the nearest pixel point. If you have recently found a root then pressing the **F0TD** key and entering the value *Root* will return the cursor to it.

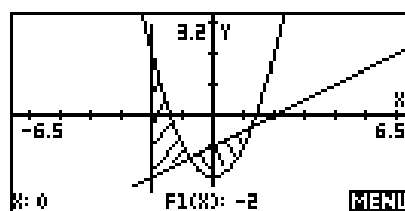
There are two ways to access these values:

- The first and simplest is via the value stored in memory X . If you move from the PLOT view to the HOME view without moving the cursor and type X then the value it will contain will be the last position of the cursor. If you just found a root or an intersection then this will be the value displayed. To find the y value for the x coordinate just evaluate $F1(X)$ in the HOME view (or whatever function you are using).
- The second way is via the reserved words of *Root*, *Extremum*, *Area*, *Slope* and *Isect*. Typing any of these reserved words in any situation will retrieve their last calculated values. If a value has not yet been found then they will return zero. You will probably only need to use those for *Area* and *Slope* since typing X will not retrieve those values.

4.5 Integration: The definite integral using PLOT variables

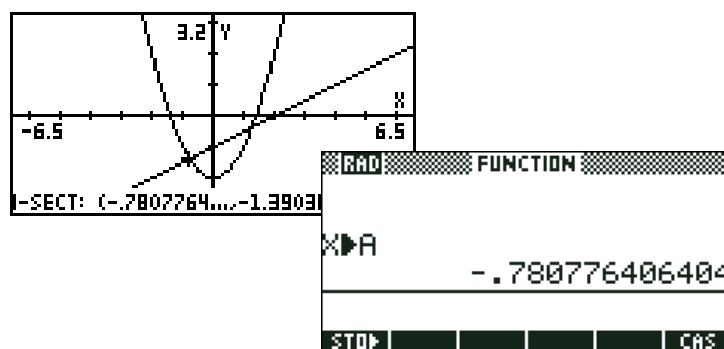
This idea of fetching FCN variables into the HOME view can be very useful in finding areas, as you will see in the example below.

Suppose we want to find the area between $f(x) = x^2 - 2$ and $g(x) = 0.5x - 1$ from $x = -2$ to the first positive intersection of the two graphs.

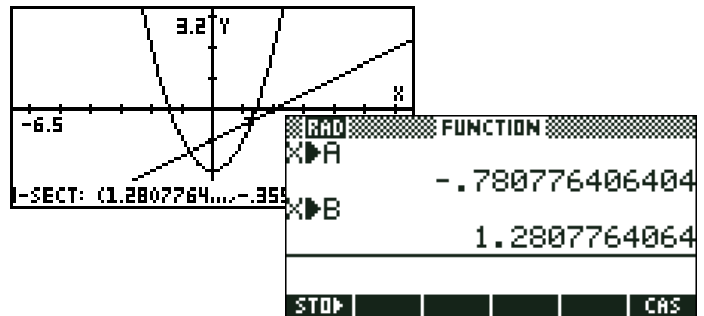


From the hand shaded screenshot shown above right it can be seen that to find the area we need to split it into two sections, with the boundaries being -2 and the two intersections.

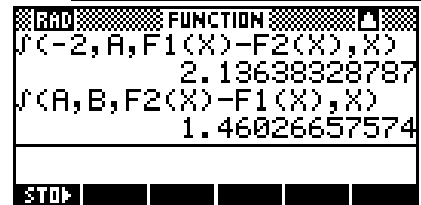
After finding the first intersection using **FCN** *Intersection* we change into the HOME view and store the results into memory variable A .



We then do the same thing for the second intersection, storing the result into B.



We can now calculate the area in the HOME view, using $f_1 - f_2$ for the first and $f_2 - f_1$ for the second. Use **COPY** to duplicate the first integral and edit it to adjust the functions and limits.



Finally, **COPY** the two solutions and add them to give the final answer.

4.6 Integration: The definite integral using the \int function

The syntax of the integration function is:

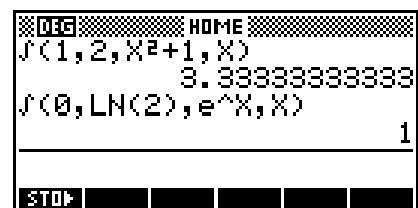
$\int(a, b, function, name)$

where: a and b are the limits of integration
and $function$ is defined in terms of $name$.

eg. $\int(1, 3, X^2 + 5, X)$ or $\int(0.5, \pi, \cos(X), X)$

The screen left shows $\int_1^2 x^2 + 1 \, dx = 3.3333\dots$

followed by $\int_0^{\ln 2} e^x \, dx = 1$.



It may help you to remember the syntax of the differentiation and integration functions if you realize that they are filled in with values in exactly the same way that they are spoken.

E.g. $\int_1^2 x^2 + 1 \, dx$ is read as:

“the integral from 1 to 2 of $x^2 + 1 \, dx$ ”

& entered the same way: $\int(1, 2, X^2 + 1, X)$

Note that:

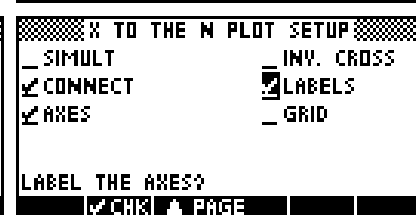
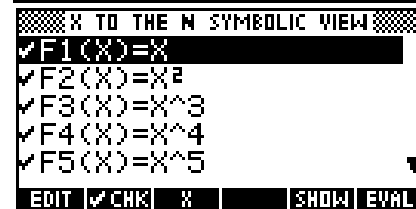
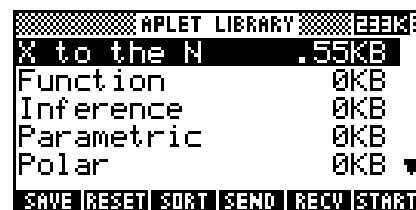
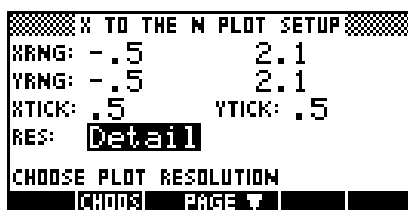
- Although I have used x as the variable of integration in the examples above, this is not a requirement. The integration example could be done as $\int (1, 2, A^2 + 1, A)$ with the same result.
- If you have defined a function in the SYMB view of the Function applet then you can use that in the HOME view as part of a calculation.
For example, as $\int (1, 3, F1(X), X)$.
- When calculating areas between curves it is easiest to do the calculations in the HOME view.
For example, as $\int (-1, 2, F1(X) - F2(X), X) + \int (2, 3, F2(X) - F1(X), X)$

4.7 Investigating $y = x^n$ for n an integer

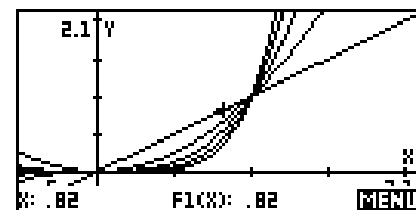
This can be done most economically by setting an investigation, perhaps for homework.

Save a copy of the Function applet under the name of "X to the N". Saving the applet will allow you to send it to your students' calculators. Alternatively you may simply choose to have the students do this themselves since it gives them practice in the skills of setting axes and saving applets.

Into this applet, enter the functions shown and set the axes as shown. The choice of x axis means that each pixel is 0.02 apart.



If you created it yourself then this applet can now be sent to each student's calculator at the end of a lesson using the IR link. Accompanying questions should address the issues below, and students should be required to either hand in a short written response, or contribute to a verbal discussion the next day.



- All the graphs have two points in common. What are they and why are they common?
- Why does an increasing value of n mean a lower value of $f(x)=x^n$ between 0 and 1, while it means a higher value for $x>1$?
- What happens for $x<0$? (The scale will need to be changed in PLOT SETUP.)
- What do the graphs of $y = x^n$ look like for negative integer values of n ?

4.8 Gradient at a Point

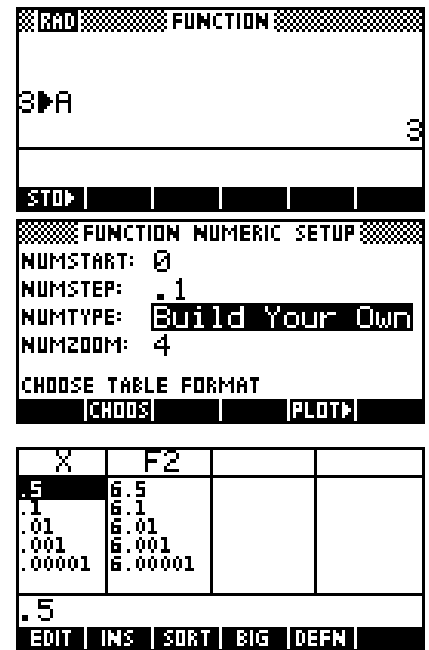
This is best introduced using an applet called “Chords” downloaded from my web site, but you can also use the Function applet.

In the Function applet, enter the function being studied into F1(X). To examine the gradient at $x=3$, store 3 into A in the HOME view as shown right, then return to the SYMB view and enter the expression shown below into F2(X).

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

Change to the NUM SETUP view and change the NumType to “Build Your Own”. You can now enter successively smaller values for X in the NUM view, since X is taking the role of h in the expression.

To investigate the gradient at a different point simply change back to the HOME view, enter a new value into A and then return to the NUM view.

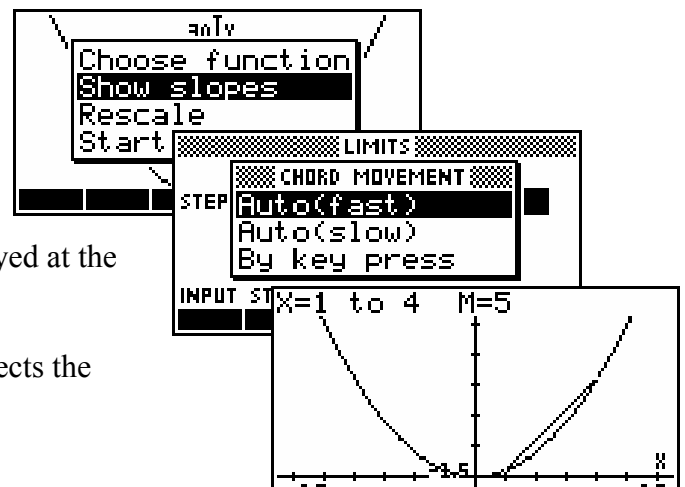


The advantage of this method is that it illustrates the limiting gradient concept without the need for tedious calculations by the student. Its disadvantage is that it is not very visual. An alternative is to use the “Chords” applet which can be downloaded from my web site.

In this applet, a menu is provided via the VIEWS key to allow students to choose from a list of predefined functions or enter their own.

Once the function has been graphed, the ‘Show slopes’ option will display an animated series of chords of diminishing length, with the gradient displayed at the top of the screen.

As the chord shortens the student can see how this affects the approximation to the gradient at the point chosen.



If it is not desirable to use this applet then the Statistics applet can be used to help with the process of finding gradient functions once tabular data has been collected giving x and $grad(x)$ values. If the student enters the data into C1 and C2, they can then set to 2VAR, plot the data and make a guess as to the appropriate function and use the curve fitting facilities to find an equation.

Curves of the form $y = mx + b$, $y = ax^2 + bx + c$, $y = ax^3 + bx^2 + cx + d$ and $y = ax^b$ can be fitted using the

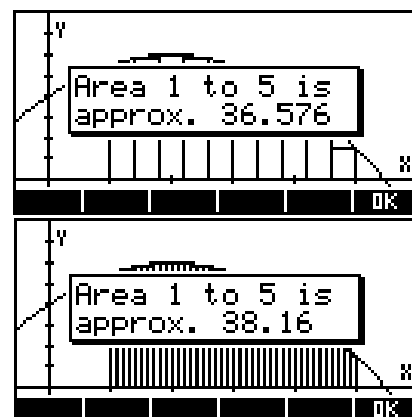
Statistics applet and this should be enough for the students to deduce the rule $\frac{d(x^n)}{dx} = n x^{n-1}$ for themselves.

It would be advisable to ensure that the students are familiar with the process of using the Statistics applet to find equations before commencing, otherwise the two concepts will interfere.

4.9 Area Under Curves

This topic is most easily handled using an applet from my web site. This applet, called “Curve Areas” will draw rectangles either over or under a curve or use trapezoids. A number of curves are supplied pre-set but the user can also enter their own. The user can nominate the interval width and the number of rectangles.

A worksheet is bundled with the applet which will lead the student through the process of deducing an area function. The worksheet asks the student to do some of the calculations themselves before using the applet.

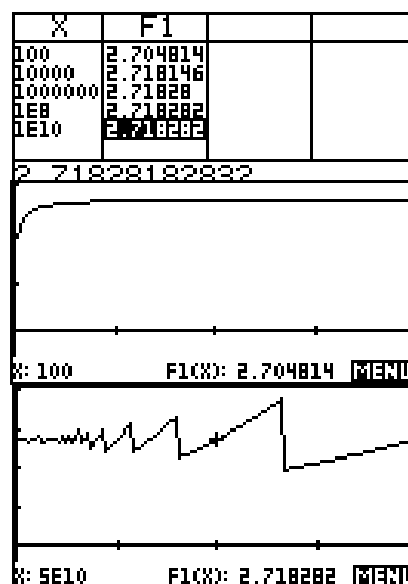


4.10 Limits

The NUM view is a very powerful tool in teaching limits. Although it does not, of course, offer a theoretical solution, it does offer a way to experimentally determine limits. However one must be careful with this since rounding error and overflow can cause incorrect results.

The first example is the determination of the value of $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$.

Use the Function applet to evaluate what happens to $F1(X) = (1 + 1/X)^X$ by changing to the NUM view and choosing the ‘Build Your Own’ facility (see right). This convergence can also be seen graphically in the PLOT view but is very slow to reach high accuracy.



The problem is that the slow convergence will mean that students will often try to graph this function for very large values of x and this can cause problems. The first graph on the right shows the expected graph for the range 0 to 100, but the second graph shows instability developing in the range 0 to $1E11$ (1×10^{11}).

This is caused by the internal rounding of the calculator. It works to 16 bits, which means that it can store 12 significant digits (for reasons only of interest to programmers). This means that when you invert a really large number and add it to one, you lose a lot of accuracy. For example: if $X = 2.85 \times 10^{10}$ then $1/X$ is $2.5087719298 \times 10^{-11}$. However, when you add 1 to this, the calculator is forced to discard all but the last decimal place. Thus $1 + 1/X = 1.00000000003$ (rounded off from $1.00000000002508\dots$)

There are naturally a whole range of numbers which will all round off to the same value of 1.00000000003 , so that (for that range of numbers) the expression $(1 + 1/X)^X$ is equivalent mathematically (on the HP) to $(1.00000000003)^X$. This produces a short section of an exponential graph, which only looks linear because you don't see enough of it.

Eventually the calculator reaches a value on the x axis which is large enough that it rounds off to a smaller number than 1.00000000003, which is 1.00000000002. This produces the sudden drop in the graph as the plot changes from a section of a 1.00000000003^X graph to a section of a 1.00000000002^X graph (which has a shallower gradient). This section is maintained then until the next drop, and so on. At the value $x = 2 \times 10^{11}$ the inverted value is so small that $1+1/X$ becomes exactly 1 and the graph will become a horizontal line (with the wrong value).

Although this explanation may be beyond the scope of the course it is quite important that students have some understanding of these ideas if they use the calculator to evaluate limits.

The second example in this section on limits illustrates the problems that can be caused by overflow. Overflow occurs when the calculator tries to store or work with a value which is larger than its capacity which, for the HP38G and HP39G, is 1×10^{500} . On the HP this value is referred to as MAXREAL and can be found in the MATH menu.

Suppose we were trying to evaluate the limit $\lim_{x \rightarrow \infty} \frac{e^x}{2e^x + 6}$. If we use a similar method of entering the function $F1(X) = e^X / (2e^X + 6)$ and using the 'Build Your Own' facility in the NUM view then an odd thing happens (see above) as the value of x increases.

X	F1		
10	.4999919		
100	.5		
500	.5		
1000	.5		
5000	.5		
10000	1		
1			
EDIT INS SORT BIG DEFN			

The reason for this is that e^x has evaluated to larger than MAXREAL. This means that the fraction is appearing to the calculator as $\frac{MAXREAL}{MAXREAL}$, which it then evaluates as equal to 1. On the HP this happens near the value $x = 1151$, which is $\ln(MAXREAL)$.

X	F1		
1000	.5		
1149	.5		
1150	.5		
1151	.7463605		
1152	1		
1			
EDIT INS SORT BIG DEFN			

4.11 Piecewise defined functions

It is possible to graph piecewise defined functions using the Function applet, although it involves literally splitting the function into pieces.

For example:

$$f(x) = \begin{cases} x+3 & ; x < -2 \\ x^2 - 2 & ; -2 \leq x \leq 1 \\ 3-x & ; x \geq 1 \end{cases}$$

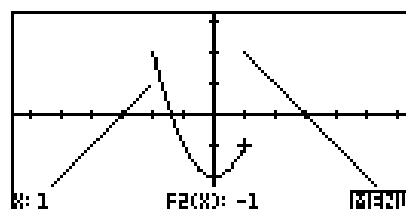
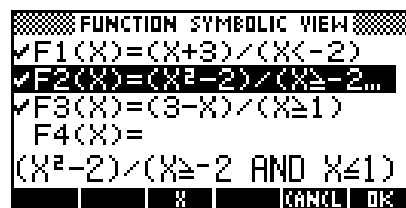
To graph this we need to enter it into the SYMB view as three separate functions:

$$F1(X) = (X+3)/(X < -2)$$

$$F2(X) = (X^2-2)/(X \geq -2 \text{ AND } X \leq 1)$$

$$F3(X) = (3-X)/(X \geq 1)$$

Note: The AND function can be found on top of the (-) key.



The reason why this works is that the $(x < -2)$ and the $(x \geq -2 \text{ AND } x \leq 1)$ expressions are evaluated as being either true (which for computers has a value of 1) or false (which has a value of 0).

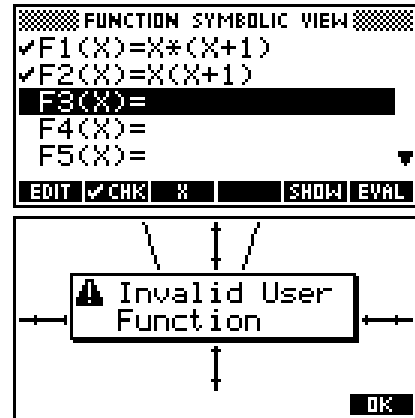
By dividing by this domain expression we are effectively dividing by 1 inside the range (with no effect) or dividing by zero outside the domain (making the function undefined). This can be seen in the NUM view to the right. Since undefined values are not graphed, this produces the desired effect.

X	F1	F2	F3
4	-1	UNDEF.	UNDEF.
2	0	UNDEF.	UNDEF.
-2	UNDEF.	2	UNDEF.
-1	UNDEF.	-1	UNDEF.
0	UNDEF.	-2	UNDEF.
1	UNDEF.	-1	2

NUM view showing values for X, F1, F2, and F3. The bottom of the screen shows a keypad with 'ZOOM', 'BIG', and 'DEFN' buttons.

4.12 Use of brackets in functions

One problem commonly encountered by new users is misinterpretation of brackets. The hp calculator will correctly interpret $F1(X) = X^2(X+1)$ as $X^2*(X+1)$ but will not understand $F(X)=X(X+1)$. When used in either Function or Solve, it will result in the error message of "Invalid User Function".



Similarly if you want to use the sum to n terms formula for a GP in the Solve aplet and enter it as $S=A(1-R^N)/(1-R)$ then you'll see a similar message unless you change it to read $S=A*(1-R^N)/(1-R)$.

The reason for this apparent 'error' is that all of the built-in functions such as SIN(...) and COS(...) and ROUND(...) work with brackets. When the calculator encounters $X(X+1)$ it interprets this as asking it to evaluate a function called $X(...)$ at the value $X+1$. Since there is no such function it returns the error message that you are trying to use a function that is unknown.

The solution is simple: just remember to put the * sign in when you use letters immediately before a bracket.

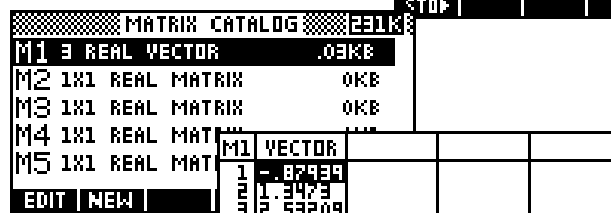
4.13 Finding and accessing polynomial roots

The POLYROOT function can be used to find roots very quickly, but the results are often difficult to see in the HOME view due to the number of decimal places spilling off the edge of the screen, particularly if they include complex roots. This can be dealt with easily by storing the results to a matrix.

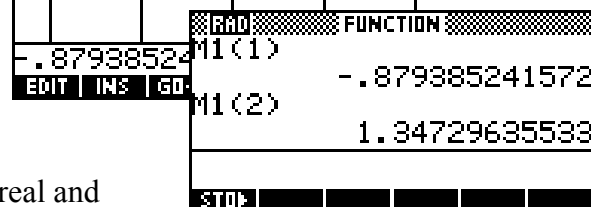
For example, suppose we want to find the roots of $f(x) = x^3 - 3x^2 + 3$. We will use the POLYROOT function and store the results into M1.



The advantage of this is that you can now view the roots by changing to the Matrix Catalog (SHIFT 4) and pressing **EDIT**.



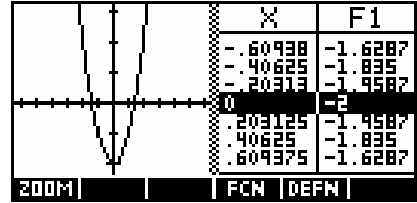
In addition to this, you can access the roots in the HOME view as shown. This method works equally well for complex roots.



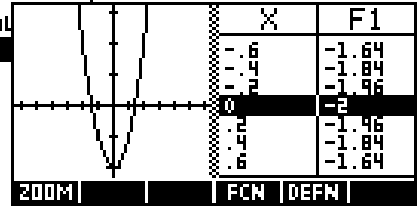
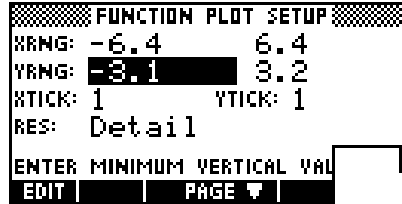
See the CAS manual for methods of finding roots of real and complex polynomials using the CAS on the hp 40gs.

4.14 Nice scales in Plot-Table

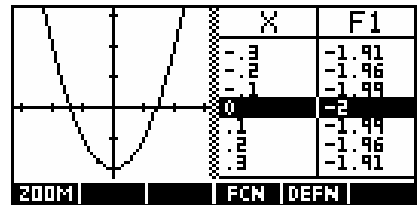
A time when 'nice' scales are more important is when you use the *Plot-Table* option in the VIEWS menu. If you use the default axes you will find that the dots, and hence the table values are no longer 'nice' because of the dots consumed by the line down the middle of the screen.



This can be solved by changing the x axis scale to -6.4 to 6.4, which gives table values of 0.2.



Using -3.2 to 3.2 is even better since it makes the graph 'square' again, with both axes proportional. Another good choice of scale for the *Plot-Table* view is -8 to 8, giving table values of 0.25. Basically any power of 2 is a good choice. Again, adding or subtracting a constant from each end of the axes will produce a graph where the y axis is not centred.



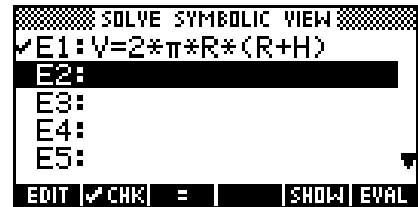
5 The Solve aplet

When there is more than one possible answer you are required to supply an initial estimate or guess. The Solve aplet will then try to find a solution which is 'near' to the estimate.

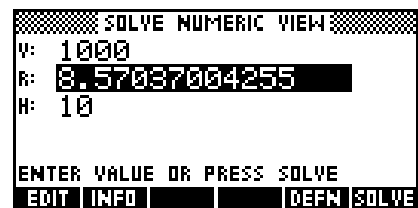
5.1 Example 1

The volume of a cylinder is given by $V = 2\pi r(r+h)$. Find the radius of a cylinder which has a volume of 1 liter and a height of 10cm.

Enter the equation into E1 as shown right. When you are entering the equation, ensure that you put a * sign between the R and the bracket.



Change to the NUM view and enter the known values, remembering that 1 liter=1000cm³. Position the highlight over R, enter a positive value as your estimate, and press **SOLVE** to find the solution shown right of 8.57cm. The equation is a quadratic in R which means two solutions are possible. If you enter an initial estimate of -10 you will obtain the negative solution, which is physically invalid in this case.

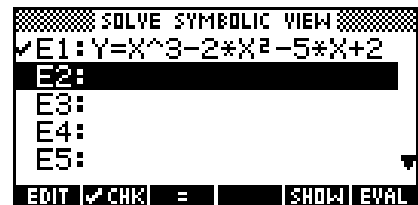


5.2 Example 2

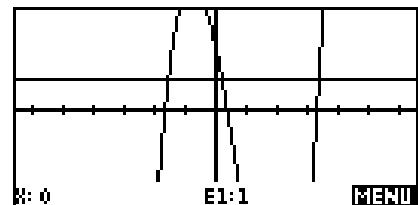
If $f(x) = x^3 - 2x^2 - 5x + 2$ find all values of x for which $f(x) = 1$.

Although you may have a clear picture in your mind and can provide Solve with the estimates it needs you would probably find it helpful to see a graph first.

It is also possible to solve this in the Function aplet, which offers more powerful tools. The PLOT view in the Solve aplet, although powerful, can be deceptive if you don't understand it and I sometimes find it easier to work in the Function aplet. In this case we will continue to work in Solve.

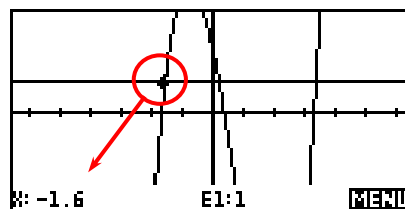


In the SYMB view, enter the equation $Y=X^3-2X^2-5X+2$ into E1. In the NUM view, enter the known value of $Y=1$, ensure that the highlight is on X, making it the active variable, and then press PLOT. If your view does not look like this then you may not have had the highlight on the X, or your axes may not be set the same as mine in PLOT SETUP.

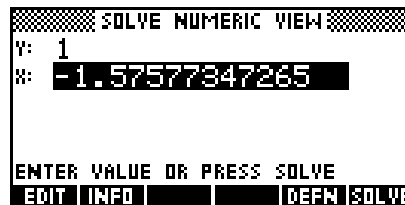


The PLOT view above shows two curves. The horizontal line is the left side of the equation which, when the known value of $Y=1$ is substituted, forms a constant straight line. The other curve is the right hand side of the equation which, since X is the active variable, forms a cubic. In this case the scale chosen is appropriate but this will not always be the case and some adjustment might be required in the PLOT SETUP view.

We require values where the two curves intersect. Using the arrow keys, move the cursor near to the first intersection point. I found (see right) that -1.6 seemed to be a good approximation. Now change back to the NUM view and you will find that this approximation has been carried back as the initial estimate. Press **SOLVE** to find the true value.



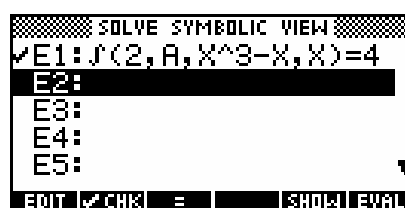
Repeat the process of obtaining an estimate in the PLOT view and refining it in the NUM view to find the other two solutions. See below for more information on the effect of the active variable on what you see in the PLOT view.



5.3 Example 3

“Find the value of a so that $\int_2^a x^3 - x \, dx = 4$ ”

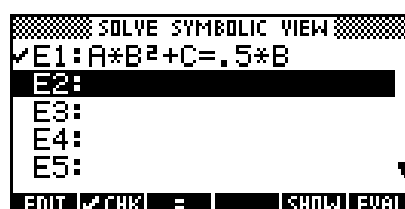
Set E1 to: $\int(2, A, X^3 - X, X) = 4$



In the NUM view, set A to an initial guess of 3, and position the highlight on A . Ignore X since it is not really involved except as a temporary variable during the integration. Press **SOLVE** to obtain an answer (eventually) of 2.4495. The delay is caused by the repeated integrations as the calculator searches for better solutions. It is important to remember that the calculator does not use algebra in Solve – it uses an algorithm which is essentially a more sophisticated version of “guess, check & improve”.

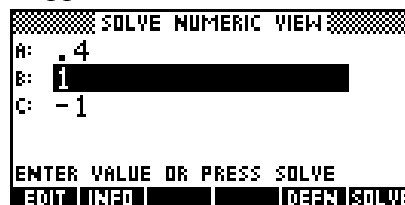
5.4 A more detailed explanation of PLOT in Solve

The PLOT view in the Solve applet is a little more complex than most others, since the active variable (x , t , θ etc) changes according to the value for which you are trying to solve.

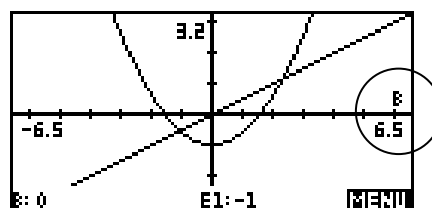


As an example, we will enter the equation $A*B^2 + C = .5*B$ into E1. Suppose that we know the values of A and C but need to find B .

Now change to the NUM view and enter the values shown right. Ensure that the highlight is on B as shown and then press PLOT.

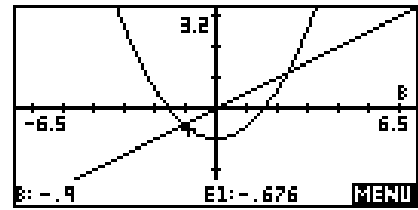


The result is a quadratic intersecting a line and the reason for this lies in how Solve interprets your equation.



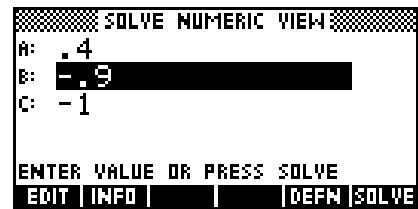
When you select B by highlighting it, the calculator substitutes the supplied values in the NUM view into all other variables except B and graphs the left and right sides of the equation as two separate graphs. This may not always be obvious because the substitution may produce graphs which aren't visible on the default scale.

In the graph above you can see from the label on the horizontal axis (circled) that the active variable is B. In this case substitution means that $A*B^2 + C = .5*B$ becomes $0.4B^2 - 1 = 0.5*B$. The left side is quadratic, while the right is linear.

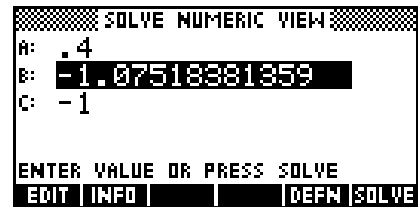


In this case there are clearly two points where the graphs intersect and hence where the left and right sides are equal.

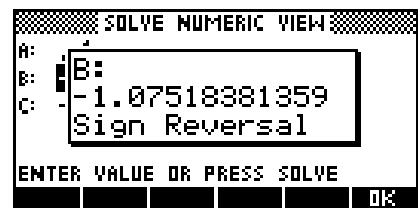
Move the cursor near to the left hand intersection and then change back to the NUM view. When you do so, the approximate value you chose with the cursor is transferred as your first 'guess'.



Now press **SOLVE** and you will see the calculator find the nearest solution to your guess. Finish by pressing **INFO** to verify that the solution is valid.

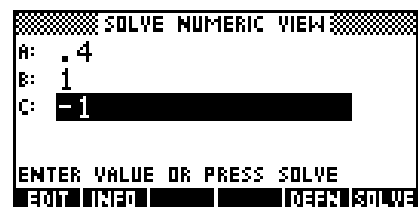


Obviously the next step is to change back to the PLOT view, move the cursor near to the second intersection and **SOLVE** for that one too.

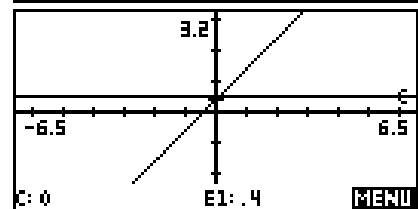


If we now change the active variable then there is an immediate change in the PLOT view to reflect this.

Change back to the NUM view and restore the variables to the original values, this time leaving the highlight on C.

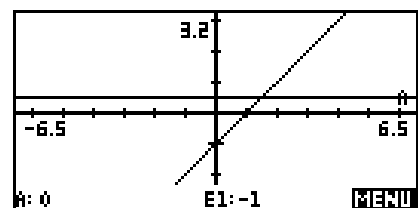


Since the active variable is now C, substitution of all other values means that $A*B^2 + C = .5*B$ becomes $0.4 + C = 0.5$. In this case both sides are linear and this is shown in the PLOT view. Notice that the axis label is now C. This would have only one solution and there would be no need to supply any particular starting value.



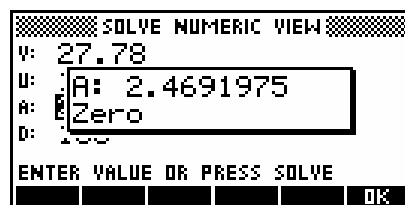
If we duplicate this process for A then substitution of values means that $A*B^2 + C = .5*B$ becomes $A - 1 = 0.5$. Both sides are still linear, although not the same linear equations as before, and this is shown in the PLOT view.

Notice that the axis label is now A.



5.5 The meaning of **INFO** messages

When you press **INFO** there are a number of possible positive responses. They are:



- **Zero** - The calculator tried to find a value of A which made this zero and, in the **INFO** message shown above, it is reporting that it succeeded. ie This is a solution.
- **Sign reversal** - This also indicates a correct solution, since normally one expects to find an answer of zero at a point where the re-arranged equation changes from positive to negative (or vice versa). 'Sign reversal' is a report that it couldn't get an answer to 12 significant digits that was precisely zero, just two answers minutely on either side of zero. This might indicate a discontinuity at the point but is more likely to indicate a satisfactory answer.

The only time that you need to worry is when you receive any of the messages below. These are:

- **Extremum** - it found a minimum, but could not reach zero. Try solving the equation $(x-2)^2 + 4 = 0$ and you will see this. The smallest value that $(x-2)^2 + 4 = 0$ can have is 4 at $x=2$, so the answer supplied will be very close to this (such as 2.000000001 or 1.999999999). The problem is that unless you check **INFO** you may not realize that this is not actually a valid solution.
- **Bad Guess** - the initial estimate you supplied was outside the domain of the function. For example, the equation uses a square root or a logarithm and you began from a value involving a negative.
- **Constant?** - no solution was found. The value of the function was the same at every point tested and wasn't the value you wanted.



Calculator Tip

It is critical that students recognize the **Extremum** case since it occurs quite often when the two sides of the equation approach closely but do not quite intersect. The student must at least recognize that the answer is invalid and, preferably, why.

This is the major reason why it is generally better to work in the Function aplet, since it allows you to see clearly whether or not there is a solution.

5.6 The Triangles applet

In the APLET view, **RESET** the Solve applet and **SAVE** it under the new name of “Triangles”. Now **START** it and enter the formulas shown.

```

TRIANGLES SYMBOLIC VIEW
E1: A^2+B^2=C^2
E2: SIN(theta)=O/H
E3: COS(theta)=A/H
E4: TAN(theta)=O/A
E5:
EDIT [CHECK] = [SHOW] EVAL
    
```

The theta character can be obtained from the keyboard on the zero button using ALPHA. Change into the MODES view and set the angle mode to *Degrees* (unless you want to use another mode). By changing into the NUM view you can now use this to solve problems in right triangles. Some users choose to use the letter D for degrees rather than theta because theta can be mistaken for an O.

5.7 The Prob. Distributions applet

In the APLET view, **RESET** the Solve applet and **SAVE** two copies of it under the new names of “Discrete PDFs” and “Cont. PDFs”.

```

APPLET LIBRARY 195K
Cont. PDFs .50KB
Discrete PDFs .50KB
Sequence .78KB
Parametric .14KB
Polar 0KB
SAVE RESET SORT SEND REC START
    
```

These applets are for use by students studying probability distribution functions, and contain formulas that can be used to perform calculations on the Binomial, Poisson, Exponential and Normal distributions. You can adapt it to the requirements of your particular course.

Firstly, **START** the **Discrete PDFs** applet and enter the formulas shown below, without the comments on the end of each one.

E1:	$V=N!/((N-X)!*X!)*P^X*(1-P)^(N-X)$; indiv. Binomial
E2:	$V=\sum_{X=S,E} (X=S,E, N!/((N-X)!*J!)*P^J*(1-P)^(N-J))$; cumul. Binomial
E3:	$P=e^{-M}*M^X/X!$; indiv. Poisson
E4:	$P=\sum_{X=S,E} (X=S,E, e^{-M}*M^X/X!)$; cumul. Poisson

Secondly, **START** the **Cont. PDFs** applet and enter the formulas shown below.

E1:	$P=1- e^{-(K*X)}$; Exponential, <X
E2:	$P=e^{-(K*X)}$; Exponential, >X
E3:	$P=e^{-(K*S)}-e^{-(K*E)}$; Exp., S<X<E
E4:	$P=1- UTPN(M, S^2, X)$; Normal, <X
E5:	$P=UTPN(M, S^2, X)$; Normal, >X
E6:	$P=UTPN(M, S^2, A)-UTPN(M, S^2, B)$; Normal, A<X<B

These formulas can be used in the NUM view to solve problems involving the probability distributions listed earlier. Some explanations and examples are given on the next page.

Note:

- S and E are used in equations E2 and E4 so that they stand for ‘start’ and ‘end’. This makes it easier for students to remember what each variable stands for in a crowded NUM view.
- The Binomial equations use the expression $N!/((N-X)!*X!)$ instead of $COMB(N,X)$ because doing this allows backward solving to solve for N. Using $COMB(N,X)$ only allows the formula to be used in a forward direction.

The drawback of this is that calculations using values of N above roughly 75 will cause overflow internally and result in errors without any sign from the calculator that this is happening. However, values like this should probably be done using a Normal approximation anyway.

If this is a problem, replace the $N!/((N-X)!*X!)$ with $COMB(N,X)$.

- Additionally, the letter v for value is used in these two equations for the binomial probability because, for this particular distribution, the variable P has the specific meaning of “the probability of individual success”.
- You may want to set both aplets to display in Fixed 5. This avoids the problem of very small probabilities appearing in Scientific Notation and being mistaken for values larger than 1.
- In most equations S and E have been used to represent ‘start’ and ‘end’. In E6 this is not possible because S is needed for the standard deviation and so A and B have been used instead. If this is a problem due to the inconsistency the you may wish to instead use the variance as below. Most people prefer to the convenience of inputting and solving for the standard deviation.

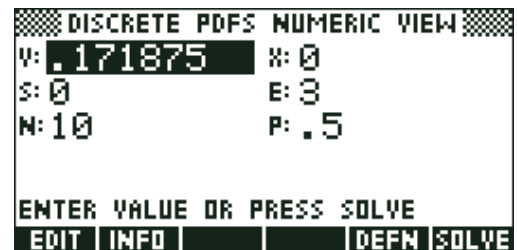
E4: $P=1-UTPN(M,V,X)$; Normal, $<X$
 E5: $P=UTPN(M,V,X)$; Normal, $>X$
 E6: $P=UTPN(M,V,S)-UTPN(M,V,E)$; Normal, $S<X<E$

Discrete PDFs: E1 and E2

These two equations can be used for calculations involving individual and cumulative Binomial probabilities.

eg. Find the probability of at most 3 heads when tossing a coin 10 times.

Ensure that formula E2 is **CHKD** and enter the values shown right. The value in J is irrelevant as it is merely the summation variable. Highlight v and press **SOLVE**.



Answer: 0.1719

Note: As mentioned earlier, if N is larger than 200 then you should use a Normal approximation. The $N!/((N-X)!*X!)$ section of the formula will cause internal overflow and inaccurate answers above this level. The time needed to perform the summation will probably also be excessive. Solve works by repeated iterations converging on the correct value and this will be quite slow if the summation has many terms. If it is important not to use a Normal approximation then replace the $N!/((N-X)!*X!)$ portion with $COMB(N,X)$. The $COMB$ function has special facilities for handling large numbers. This will not help greatly with the iteration time needed.

Discrete PDFs: E3 and E4

These two equations can be used for calculations involving individual and cumulative Poisson probabilities, where M is the mean. The technique is otherwise identical to the Binomial problem.

Cont. PDFs: E1, E2 and E3

These equations give $P(x \leq X)$, $P(x \geq X)$ and $P(S \leq x \leq E)$ for an Exponential distribution with parameter $k = 1/\mu$, where S and E stand for 'start' and 'end'.

Note: As an alternative you may wish to use $1/M$ instead of K so that you don't need to invert the mean. In this case equation 1 would become E1: $P = 1 - e^{-(1/M * X)}$ and so on.

Cont. PDFs: E4, E5 and E6

These equations give $P(x \leq X)$, $P(x \geq X)$ and $P(S \leq x \leq E)$ for a Normal distribution with parameters mean = M and variance = V . Some people prefer to use the standard deviation S and in this case the equations would need to be:

6 Miscellaneous tricks in other applets

6.1 Rectilinear Motion

Rectilinear motion can be enhanced by using the Parametric applet to graphically illustrate the motion of a particle. If this is set up properly then it can be a very helpful teaching aid, as the graph will slow down and speed up as it appears, illustrating the velocity and acceleration of the particle.

For example, consider the particle with motion equation $x(t) = t^3 - 8t^2 + 17t - 10$. Suppose that we are interested in the first six seconds of its path. We enter this equation as X1(T) in the Parametric applet and enter Y1(T)=0.1T as its companion.

```
PARAMETRIC SYMBOLIC VIEW
✓X1(T)=T^3-8*T^2+17...
✓Y1(T)=.1*T
X2(T)=
Y2(T)=
X3(T)=
EDIT ✓CHK| T | SHOW| EVAL
```

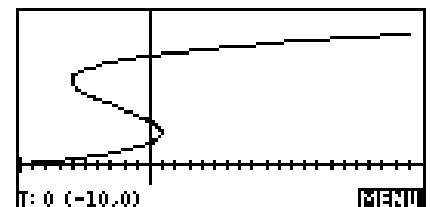
The reason for the Y1(T) equation is firstly because the Parametric equation requires paired equations, and secondly so as to spread the motion up the y axis as the particle moves. The idea is to have the particle reach the top of the screen at the end of the time period being examined so choose the maximum value on your y axis accordingly. If you're examining the first 8 seconds, for example, then using Y1(T)=0.1T will mean that the largest y value will be 0.8. Consequently setting the y axis to (say) -0.3 to 1 would be a good choice.

```
PARAMETRIC PLOT SETUP
TRNG: 0 6
TSTEP: .05
XRNG: -10 21
YRNG: -.2 .7
ENTER MINIMUM TIME VALUE
EDIT PAGE ▾
```

In this case above, examination of the NUM view will show that an x axis range of -10 to 21 will cover the required T range and, for a T range of 0 to 6, a y axis range of -0.2 to 0.7 will spread it adequately.

The value of TStep determines how fast the particle 'moves' and a value of 0.05 is usually a good choice.

The result is shown right but it needs to be seen as it is being drawn to appreciate how the particle slows down and speeds up as it passes the turning points. Try it and see.



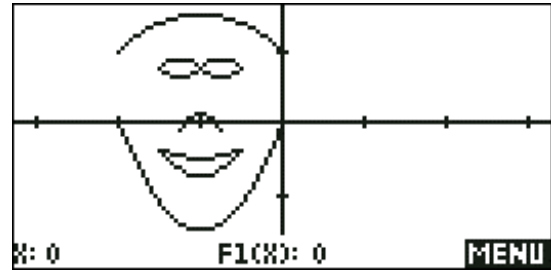
6.2 Having fun with graphs

A good way to familiarize students with the calculator is to have them draw ‘pictures’ or patterns using the aplets. In Function, IF you have a reasonably able class, you can use the idea of piecewise defined functions and offer a prize to the best picture produced. Give them a few days to a week to work on them as they will need it to produce a really excellent result.

This use of the piecewise defined function also enhances the students’ understanding of the concept of the domain. Using restricted domains, with only quadratics and lines, it is quite easy to draw pictures such as the one shown right.

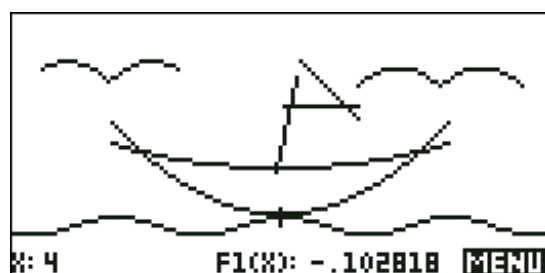
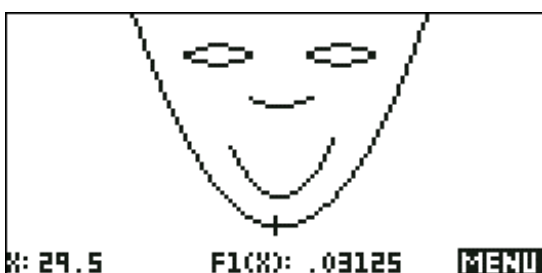
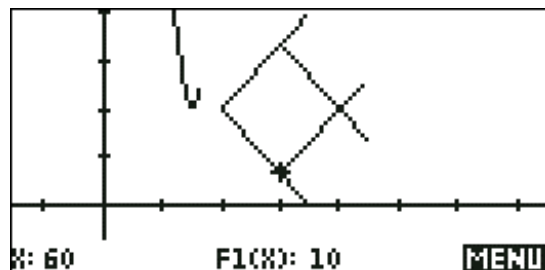
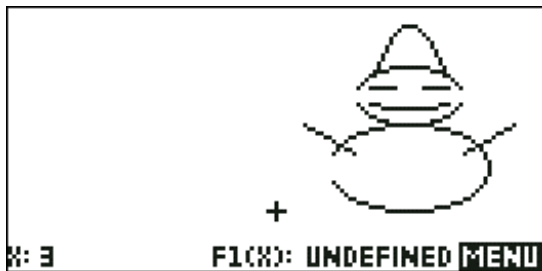
I begin the activity by displaying the image above as a ‘teaser’ and then explaining that it was entirely built using quadratics. I show the students the equations that I have used (see fig. 1) and explain how the calculator can be used to display restricted domains by dividing by the domain in the form of a logical expression.

The next stage of the activity is to challenge the group to produce their own images. They were given a week to work and in this case I placed two restrictions on their work. The first was that they must use no more than 10 functions to form the image and the second was that they must restrict themselves to quadratics and linear functions. The incentive of a chocolate prize was added in order to start the students off with enthusiasm! Some of the results are shown below. As can be seen, the complexity of the resulting images varied greatly.



$$\begin{aligned}
 f_1(x) &= 0.75(x+2)^2 - 3; & -4 \leq x \leq 0 \\
 f_2(x) &= 0.25(x+2)^2 - 1; & -3 \leq x \leq -1 \\
 f_3(x) &= -0.25(x+2)^2 + 3; & -4 \leq x \leq 0 \\
 f_4(x) &= -2(x+2)^2 + 0.25; & -2.5 \leq x \leq -1.5 \\
 f_5(x) &= -(x+1.5)^2 + 1.75; & -2 \leq x \leq -1 \\
 f_6(x) &= (x+1.75)^2 + 1.25; & -2 \leq x \leq -1 \\
 f_7(x) &= -(x+2.5)^2 + 1.75; & -3 \leq x \leq -2 \\
 f_8(x) &= (x+2.5)^2 + 1.25; & -3 \leq x \leq -2 \\
 f_9(x) &= 0.75(x+2)^2 - 1.5; & -3 \leq x \leq -1
 \end{aligned}$$

Fig. 1



In addition to this the Polar and Parametric aplets can be used to produce interesting graphs, even with students who don't really understand what a polar or parametric equation is.



Show them this image, made up of the functions below and simply challenge them to enter the equations into the Polar aplet themselves and then experiment. You may be surprised at what results.

```

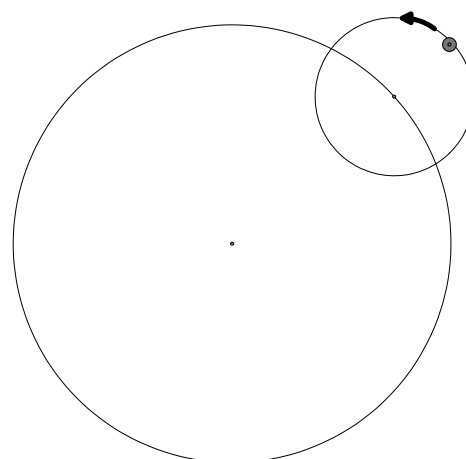
POLAR SYMBOLIC VIEW
✓R1(θ)=3
✓R2(θ)=3*SIN(10*θ)
✓R3(θ)=3+ABS(COS(10*θ))
R4(θ)=
R5(θ)=
EDIT ✓CHK 0 SHOW EVAL
  
```

Note that you must change the value of θ Step in the PLOT SETUP view to 0.05 rather than the default value of $\pi/24$. The default value will give a very jagged graph as shown right. Some students find that this jagged result is more interesting!



The Parametric graph can be used to create fascinating graphs. If you're teaching the Unit Circle and you've covered the fact that the x and y coordinates are given by $r \cdot \cos\theta$ and $r \cdot \sin\theta$ respectively then you can, with a more advanced class, cover the idea of building a spirograph using Parametric.

Imagine following the path of the dot shown on the diagram as it rotates around the small circle while at the same time the centre of the small circle rotates rapidly around the large one.



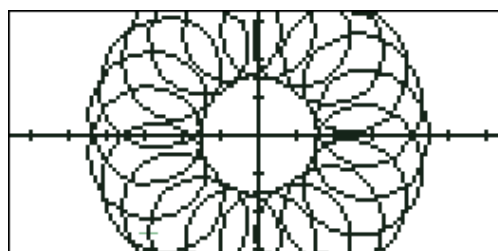
The path can be modeled using the equations:

$$X1(T)=3 \cdot \cos(T)+1.5 \cdot \cos(20 \cdot T)$$

$$Y1(T)=3 \cdot \sin(T)+1.5 \cdot \cos(20 \cdot T)$$

These equations are set to assume that the radius of the large circle is 3, of the small one is 1.5 and that the smaller one is rotating at 20 times the rate that its centre is traveling around the large one.

The result is shown right. It should be noted that this requires some changes in the PLOT SETUP view. Change T_{\max} to 2π and T_{Step} to 0.02. If you don't do this then the circle will rotate more than once and the graph will be jagged.

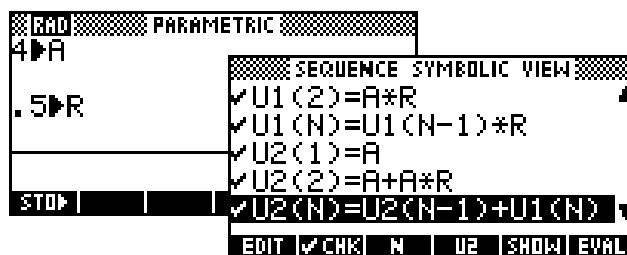


Changing the coefficients in the equations above results in some very interesting graphs and there are relationships between coefficients as to what type of graph is produced based on whether they have common factors and on relative sizes. Try changing the 1.5 to 1.2 and the 20 to 2 for example. Try changing the radius of the second circle so that it has a radius larger than the main one. Try experimenting with slower rotations.

6.3 Sequences and Series

Through the Sequence applet the HP provides very flexible tools for the investigation of sequences. These can also be adapted to investigate series as well. We will look at geometric sequences but the same methods will also work for arithmetic sequences.

Begin by storing the values 4 and 0.5 into A and R respectively in the HOME view (see right). These values will be changed later.



In the Sequence applet, enter the general sequence with:

$$\begin{aligned} U1(1) &= A \\ U1(2) &= A * R \\ U1(N) &= U1(N-1) * R \end{aligned}$$

then the general series with:

$$\begin{aligned} U2(1) &= A \\ U2(2) &= A + A * R \\ U2(N) &= U2(N-1) + U1(N) \end{aligned}$$

If you are concerned that your students will not follow the use of A and R to generalize the sequence and series then just enter the equations using specific values.

If you now change to the NUM view then the values of the sequence and its accompanying series will be visible. Scrolling down through the values will make it obvious that the series is converging on the value 8.

N	U1	U2
1	4	4
2	2	6
3	1	7.5
4	.5	7.75
5	.25	7.875

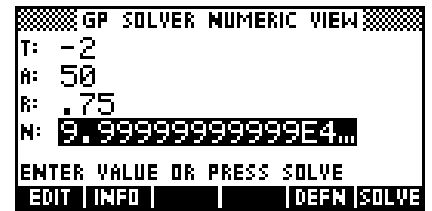
Returning to the HOME view, the student can now adjust the values of A and/or R to obtain any geometric sequence or series. If you SAVE this applet under the name (say) of “GP Sequence” then you can re-use this later.

The teacher might create a worksheet to take the student deducing the formula for the sum to infinity through recording his or her results as A and R change.

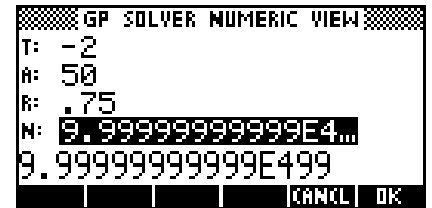
Once the formulae for the sequence and series have been covered an excellent tool can also be created by making a copy of the Solve applet and entering the appropriate formulae.

In the APLET view, COPY the Solve applet and name it “GP Solver”. In the SYMB view enter the three formulae $T = A * R^{(N-1)}$, $S = A * (1 - R^N) / (1 - R)$ and $S = A / (1 - R)$. These can now be used to solve for any one of the terms they contain given the others.

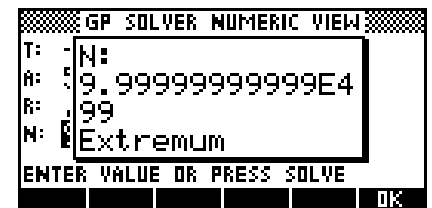
Note: One of the common errors that students make in the Solve applet is to try to solve for solutions which do not exist. For example, a student will try to solve for N in $T=A \cdot R^{(N-1)}$ with $A=50$, $R=0.75$ and $T=-2$. The result will firstly be that the calculator will seem to freeze while it tries to find a solution which can only be approached asymptotically.



Finally the calculator will give the result as shown right. The problem is that students will often misinterpret it as being $N=10$, when in fact it is simply that the calculator has gone as far along the positive x axis as possible and stopped at MAXREAL of 1×10^{500} (see second screen shot).



It is recommended that the teacher should deliberately provoke this error and follow with class discussion. Students might be encouraged to press the INFO button after finding a solution since a case like this will give 'Extremum' whereas a correct solution will result in either 'Zero' or 'Sign Reversal' as the algorithm zero's in on the answer by looking for an exact match (Zero) or a close one (Sign change).



6.4 Modelling a loan

Modelling a loan or annuity can be done fairly easily in the Sequence applet.

Investment in an interest bearing account.

This is relatively easy to model using one sequence but can be enhanced by using two. Suppose we invest \$2000 at 5% pa compounded annually. We would use...

SEQUENCE SYMBOLIC VIEW

✓U1(1)=2000
U1(2)=
✓U1(N)=U1(N-1)*1.05
U2(1)=
U2(2)=

EDIT ✓CHK SHOW EVAL

N	U1		
1	2000		
2	2100		
3	2205		
4	2315.25		
5	2431.013		
6	2552.563		

We can enhance this by making things a little more general as below. This means that students can simply change the values of P and R without having to re-enter the equations.

RAD SEQUENCE

2000▶P
1.05▶R

STO▶ CAS

SEQUENCE SYMBOLIC VIEW

✓U1(1)=P
U1(2)=
✓U1(N)=U1(N-1)*R
U2(1)=
U2(2)=

EDIT ✓CHK SHOW EVAL

If the interest calculations are being made bi-annually then just use $1+0.05/2$ as R and regard each term as six months. Similarly for monthly.

A further enhancement is to add a date column. Suppose that the initial deposit of \$P was done in 1998 and you want to know the value of the money in the account in 2010. You can, of course count down to 12... or is it 11 or 13...? Instead, add a second sequence showing the year. You may find, as I did, that the calculator requires you to input the second term before it will tick/check the sequence. This is a bug.

N	U1	U2	
9	2954.911	2006	
10	3102.656	2007	
11	3257.789	2008	
12	3420.629	2009	
13	3591.719	2010	
14	3771.248	2011	
3591.71265205			
ZOOM			BIG DEFN

A housing loan is probably the next most common situation and can easily be modeled in a similar way. The trick is not to try to put it all in one sequence, since that means no working will be shown. Ideally we'd like it to produce something similar to the table of values below in the NUM view.

If we begin with a table of values, this gives us both the first two terms and some values to check with. We'll borrow \$20,000 in 2006 at 5% pa compound interest calculated annually with one repayment of \$2000 made annually. The questions are, how long to repay the loan and what is the total repayment?

Year	Initial Balance	Interest @ 5%	Repayment	Final Balance
0	\$20,000	\$20,000x1.05 = \$21,000	\$2000	\$19,000
1	\$19,000	\$19950	\$2000	\$17,950
2	\$17,950	\$18,847.50	\$2000	\$16,847.50
3	\$16,847.50	\$17,689.88	\$2000	\$15,689.88

In this example we will again try to make it more general by using P for the Principal borrowed, R for the interest rate multiplier and D for the repayment/deposit.

N	U1	U2	
2006	20000		
2007	19000		
2008	17950		
2009	16847.5		
2010	15689.88		
2011	14474.37		
2009			
ZOOM			BIG DEFN

Scrolling down, together with some further calculations allows us to answer the questions.

6.5 Problems with the User Defined fit model

As soon as the line of best fit has been obtained, you can switch back to the SYMB view to get its equation. Note that this equation is only calculated when called for by pressing the **FIT** key.

In the SYMB view (see right) the equation is given to so many decimal places that it doesn't fit onto the screen. The simplest way to see the entire equation is to position the highlight on the equation and press the **SHOW** key.

```

STATISTICS SYMBOLIC VIEW
✓S1: C1      C2
✓Fit1: .819944598333...
S2:
Fit2: m*X+b
ENTER USER DEFINED FIT
EDIT ✓CHK X SHOW EVAL

```

When you do you will obtain a view similar to the one seen right but with more decimals. You can scroll right and left to view the equation.

```

0.8199*X+1.1662
EDIT ✓CHK X SHOW EVAL

```

On the screen above the equation is given to 4 decimal place accuracy. This is accomplished by changing to the MODES screen (above the HOME key) and specifying Fixed 4 as the Number Format. If you don't do this then the alternative is to use the arrow keys to scroll.

```

HOME MODES
ANGLE MEASURE: Radians
NUMBER FORMAT: Fixed 4
DECIMAL MARK: Dot(.)
CHOOSE FORMAT FOR NUMBERS
[CHOOSE]

```



Calculator Tip

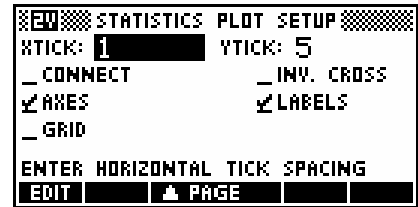
One of the most common mistakes people make is to press **EDIT** instead of **SHOW**. Always use **SHOW** when you want to display the entire equation.

It is very important that you display the equation in the SYMB view by pressing **EDIT** instead of **SHOW**. Although pressing **EDIT** displays the equation in a form which can be scrolled through, the problem with this method is that the calculator thinks that by editing it you are choosing to override its choice of equation with your own. As long as you press **CANCEL** there is no problem but if you press ENTER it changes the FitType from whatever you had previously chosen to *User Defined* instead.

If you are not aware that this has happened then it can have very serious consequences. For example, if you were now to change the data and then re-plot, the fit equation will not be re-calculated and will become inaccurate. Any predictions made from it would be incorrect.

6.6 Connected data

One of the settings on the second page of the PLOT SETUP screen can be useful for some types of data. For example, one of the common tasks in many mathematical courses is the analysis of time series data. Unlike most bivariate data, time series values are usually plotted as a line graph - i.e. as connected points. This facilitated by Connect.



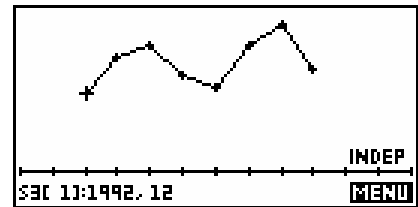
For example:

The sales of an ice-cream shop are shown as quarterly sales figures for the years 1992 to 1993. Display this data on a line graph.

Yr Qtr	1992				1993			
	1	2	3	4	1	2	3	4
Sales (\$1000)	12	18	20	15	13	20	23	16

The result (with Connect checked) is shown right. Notice that although the x values in the table were 1, 2, 3, 4, 1, 2, 3, 4 we can't use them that way because it would result in 1992 Qtr 1 being graphed on top of 1993 Qtr 1.

The graph on the right was produced by entering values of 1992, 1992.25, 1992.5, 1992.75 etc.

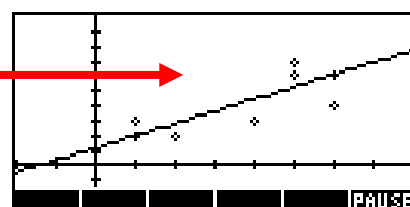
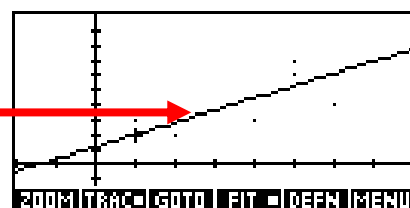
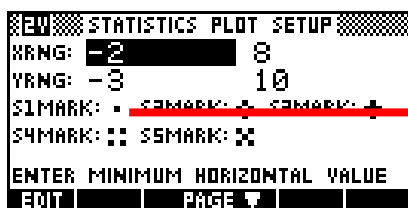


This is not the end of the story though. When you activate Connect the calculator will always sort the points into ascending order based on their x values before graphing. Thus (1,3) (5,4) (2,7) would be graphed as (1,3) (2,7) (5,4). Not doing this would result in a graph that doubled back on itself and looked like a scribble instead of a line graph. A better alternative to this process is to download the *Time Series* applet from the website **The HP Home View** at www.hphomeview.com. This completely analyses the data.



Calculator Tip

If you have trouble seeing the small dots that the calculator uses in its scatter-graphs by default then you can change the S1Mark. Highlight the mark for the data set you are using, press **CHODS** and you will see the menu shown below from which you can choose a different mark. The contrast is illustrated below.



6.7 Finding and plotting residuals

Supposing that you have the data in columns C1 (x) and C2 (y) then the first step is to PLOT the data and FIT the curve to it. Until this is done the residual equation has not been calculated and can't be used to create residuals.

Now change into the HOME view and calculate the residuals using $C2 - \text{PREDY}(C1) \rightarrow C3$. This calculates them by performing the real-predicted calculation on the entire column at once, storing the result in to the new column C3.

If you now change into the SYMB view you can un-tick the original data C1 vs C2 and enter the residuals as C1 vs C3. Using VIEWS – Auto Scale will generally then produce a good result.